Name:

Spring 2017 Math 245 Final Exam

Please read the following directions:
Please write legibly, with plenty of white space. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may use a single page of notes, but no calculators or other aids. This exam will last 120 minutes; pace yourself accordingly. Please try to keep a quiet test environment for everyone. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
11.	5		10
12.	5		10
13.	5		10
14.	5		10
15.	5		10
16.	5		10
17.	5		10
18.	5		10
19.	5		10
20.	5		10
Total:	100		200

Problem 1. Carefully define the following terms:

a. Division Algorithm theorem

b. contradiction (proposition)

c. Vacuous Proof theorem

d. predicate

Problem 2. Carefully define the following terms:

a. Nonconstructive Existence Proof theorem

b. Fibonacci number sequence

c. big O

d. List notation (for sets)

Problem 3. Carefully define the following terms:

a. Cartesian (direct) product

b. (binary) relation

c. irreflexive

d. modular equivalence relation

Problem 4. Carefully define the following terms:

a. equivalence class

b. poset (partially ordered set)

c. Sperner's theorem

d. image

Problem 5. Find the converse of the contrapositive of $q \to (p \vee r)$.

Problem 6. Let S be a set. Prove that $\emptyset \subseteq S$.

Problem 7. Find all integers x with $0 \leq x < 24$, that satisfy $9x \equiv 18 \pmod{24}$.

Problem 8. Let S be a set. Prove that $S_{diagonal}(= id_S)$ is an equivalence relation.

Problem 9. Let $n \in \mathbb{Z}$. Use the definitions of even and odd to prove that n cannot be both even and odd.

Problem 10. Let p, q be propositions. Use a truth table to prove that $p \oplus q \equiv (p \wedge \neg q) \vee (q \wedge \neg p)$.

Problem 11. Let $x \in \mathbb{R}, k \in \mathbb{Z}$. Prove that $\lfloor x + k \rfloor = \lfloor x \rfloor + k$.

Problem 12. Prove that, for all $n \in \mathbb{N}$, $\sum_{i=1}^{n} (-1)^i i^2 = \frac{n(n+1)(-1)^n}{2}$.

Problem 13. Prove that $n^2 + 7 = \Omega(n^2)$.

Problem 14. Let S, T, U be sets with $S \subseteq U, T \subseteq U$. Prove that $S^c \cup T^c \subseteq (S \cap T)^c$.
Note: This is part of De Morgan's Law for Sets. Do not use this theorem to prove itself.

For problems 15-17, let R be the "divides" relation, i.e. $(a, b) \in R$ if $a|b$.

Problem 15. Let R be as above, on $\mathbb{N}$. Prove that R is transitive.

Problem 16. Let R be as above, restricted to $S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. Find all maximal and minimal elements on $T = S$. You may assume that R is a partial order.

Problem 17. Let R be as above, restricted to $S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. You may assume that R is a partial order. Find the width and height of this poset.

Problem 18. Let S be a set, and R a relation on S. Prove that $R \cup R^{-1}$ is symmetric.

Problem 19. Consider $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \sin x$. Determine whether or not f is surjective. Also, determine whether or not f is injective. Be sure to justify your answers.

Problem 20. Let $\mathbb{R}^{\geq 1} = \{x \in \mathbb{R} : x \geq 1\}$. Define a relation $R \subseteq \mathbb{R}^{\geq 1} \times \mathbb{N}_0$ as follows: $R = \{(x, n) : 2^n \leq x < 2^{n+1}\}$. Prove that R is well-defined, i.e. R is a function.